

CBCS SCHEME

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BMATS201

Second Semester B.E./B.Tech. Degree Examination, June/July 2025 Mathematics II for CSE Stream

Time: 3 hrs.

Max. Marks: 100

- Note:** 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. VTU Formula Hand Book is permitted.
3. M : Marks, L: Bloom's level, C: Course outcomes.

| Module - 1 | | | | M | L | C |
|------------|----|---|--|----|----|-----|
| Q.1 | a. | Evaluate $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x+y+z) dy dx dz$ | | 07 | L3 | CO1 |
| | b. | Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy dx$ by changing the order of integration. | | 07 | L3 | CO1 |
| | c. | Show that $\beta(m,n) = \frac{\Gamma(m) \cdot \Gamma(n)}{\Gamma(m+n)}$ | | 06 | L2 | CO1 |
| OR | | | | | | |
| Q.2 | a. | Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2) dy dx$ by changing to polar coordinates. | | 07 | L3 | CO1 |
| | b. | Using double integration find the area of a plate in the form of a quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ | | 07 | L3 | CO1 |
| | c. | Using mathematical tools, write the code to find the volume of the tetrahedron bounded by the planes $x=0$, $y=0$, $z=0$ and $6x+3y+2z=6$. | | 06 | L3 | CO5 |
| Module - 2 | | | | | | |
| Q.3 | a. | Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$. | | 07 | L2 | CO2 |
| | b. | Evaluate $\text{div } \vec{F}$ and $\text{curl } \vec{F}$ at the point $(1, 2, 3)$, given $\vec{F} = \text{grad}(x^3y + y^3z + z^3x - x^2y^2z^2)$ | | 07 | L3 | CO2 |
| | c. | Express the vector $\vec{F} = 2x\hat{i} - 3y^2\hat{j} + zx\hat{k}$ in cylindrical form. | | 06 | L3 | CO2 |
| OR | | | | | | |
| Q.4 | a. | Find the directional derivative of $\phi(x, y, z) = x^2yz + yxz^2$ at $(1, -2, 1)$ in the direction of the vector $2\hat{i} - \hat{j} - 2\hat{k}$. | | 07 | L2 | CO2 |
| | b. | Show that the vector $\vec{F} = \frac{x\hat{i} + y\hat{j}}{x^2 + y^2}$ is both solenoidal and irrotational. | | 07 | L3 | CO2 |
| | c. | Using mathematical tool, write the code to find the gradient of $xy^3 + yz^3$. | | 06 | L3 | CO5 |

Module – 3

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|-----|----|--|----|----|-----|
| Q.5 | a. | Show that the set $W = \{(x, y, z) / x + y + 2z = 0\}$ of the vector space $V_3(R)$ is a subspace of $V_3(R)$. | 07 | L2 | CO3 |
| | b. | Find the basis and dimension of the subspace spanned by the vectors $\{(1, -1, 0), (0, 3, 1), (1, 2, 1), (2, 4, 2)\}$ in R^3 . | 07 | L3 | CO3 |
| | c. | Find the matrix of the linear transformation $T : V_2(R) \rightarrow V_3(R)$ such that $T(-1, 1) = (-1, 0, 2)$ and $T(2, 1) = (1, 2, 1)$. | 06 | L2 | CO3 |

OR

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|-----|----|--|----|----|-----|
| Q.6 | a. | Determine whether the following set of vectors in 2×2 matrix space is linearly independent or linearly dependent: $S = \{v_1, v_2, v_3\}$ where $v_1 = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}$ and $v_3 = \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}$ | 07 | L2 | CO3 |
| | b. | Prove that $T : R^3 \rightarrow R^2$ defined by $T(x, y, z) = (x + y, y + 2z)$ is a linear transformation. | 07 | L2 | CO3 |
| | c. | Verify the Rank – nullity theorem for the linear transformation $T : R^3 \rightarrow R^2$ defined by $T(x, y, z) = (y - x, y - z)$. | 06 | L2 | CO3 |

Module – 4

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|------------------|--------|--|-------|-------|-------|----|----|----|------------------|--------|-------|-------|-------|-------|----|----|-----|
| Q.7 | a. | Find a real root of the equation $x \log_{10} x = 1.2$ by regula falsi method corrected to 4 decimal places between (2.5, 3). Carry out 4 iterations. | 07 | L2 | CO4 | | | | | | | | | | | | |
| | b. | From the following table of half yearly premium for policies maturing at different ages, estimate the premium for the policies maturing at the age of 46. <table border="1" data-bbox="316 1279 1062 1357"> <tr> <td>Age :</td><td>45</td><td>50</td><td>55</td><td>60</td><td>65</td></tr> <tr> <td>Premium (in Rs.)</td><td>114.84</td><td>96.16</td><td>83.32</td><td>74.48</td><td>68.48</td></tr> </table> | Age : | 45 | 50 | 55 | 60 | 65 | Premium (in Rs.) | 114.84 | 96.16 | 83.32 | 74.48 | 68.48 | 07 | L3 | CO4 |
| Age : | 45 | 50 | 55 | 60 | 65 | | | | | | | | | | | | |
| Premium (in Rs.) | 114.84 | 96.16 | 83.32 | 74.48 | 68.48 | | | | | | | | | | | | |
| | c. | Use Trapezoidal rule to estimate the integration $\int_0^1 \frac{1}{1+x^2} dx$ by taking 10 equal intervals. | 06 | L3 | CO4 | | | | | | | | | | | | |

OR

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|--------|------|---|-----|----|-------|---|---|---|--------|------|----|---|---|-------|----|----|-----|
| Q.8 | a. | Find by Newton's Raphson method, the root of the equation $\cos x = x e^x$ near to 0.5, corrected to 4 decimal places. | 07 | L2 | CO4 | | | | | | | | | | | | |
| | b. | Using Newton's divided difference interpolation, find the interpolating polynomial of the given data: <table border="1" data-bbox="316 1738 831 1816"> <tr> <td>x :</td><td>-4</td><td>-1</td><td>0</td><td>2</td><td>5</td></tr> <tr> <td>f(x) :</td><td>1245</td><td>33</td><td>5</td><td>9</td><td>+1335</td></tr> </table> | x : | -4 | -1 | 0 | 2 | 5 | f(x) : | 1245 | 33 | 5 | 9 | +1335 | 07 | L3 | CO4 |
| x : | -4 | -1 | 0 | 2 | 5 | | | | | | | | | | | | |
| f(x) : | 1245 | 33 | 5 | 9 | +1335 | | | | | | | | | | | | |
| | c. | Evaluate using Simpson's $(1/3)^{rd}$ rule, $\int_4^{5.2} \log x dx$ by dividing the range (4, 5.2) into seven ordinates. | 06 | L3 | CO4 | | | | | | | | | | | | |

Module – 5

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|------|----|---|--------|--------|-----|-----|-----|-----|---|--------|--------|--------|----|----|-----|
| Q.9 | a. | Solve $y' = 3x + y^2$; $y(0) = 1$ using Taylor's series method at $x = 0.1$ and $x = 0.2$. | 07 | L3 | CO4 | | | | | | | | | | |
| | b. | Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with $y = 1$ when $x = 0$. Find an approximate y at $x = 0.1$ using Euler's modified method. (Use modified formula twice). | 07 | L3 | CO4 | | | | | | | | | | |
| | c. | From the data given below find y at $x = 1.4$ using Milne's method. Given $\frac{dy}{dx} = x^2 + \frac{y}{2}$. <table border="1"><tr><td>x :</td><td>1</td><td>1.1</td><td>1.2</td><td>1.3</td></tr><tr><td>y :</td><td>2</td><td>2.2156</td><td>2.4549</td><td>2.7514</td></tr></table> | x : | 1 | 1.1 | 1.2 | 1.3 | y : | 2 | 2.2156 | 2.4549 | 2.7514 | 06 | L3 | CO4 |
| x : | 1 | 1.1 | 1.2 | 1.3 | | | | | | | | | | | |
| y : | 2 | 2.2156 | 2.4549 | 2.7514 | | | | | | | | | | | |
| OR | | | | | | | | | | | | | | | |
| Q.10 | a. | Using Runge – Kutta method of order 4, find y at $x = 0.2$, given $\frac{dy}{dx} = \frac{x^2 + y^2}{10}$; $y(0) = 1$ taking $h = 0.1$. | 07 | L3 | CO4 | | | | | | | | | | |
| | b. | Using modified Euler's method solve $y' = 3x + \frac{y}{2}$ with $y(0) = 1$ taking $h = 0.2$ at $x = 0.2$. (Use modified Euler's formula twice). | 07 | L3 | CO4 | | | | | | | | | | |
| | c. | Using mathematical tools, write the code to solve $\frac{dy}{dx} = x^2y - 1$; $y(0) = 1$ by Taylor's series method at $x = 0.1$ (0.1) 0.3 | 06 | L3 | CO5 | | | | | | | | | | |
